

✓ 5-Minute Check

Over Lesson 8-6

3 Solve $y^2 - 8y - 20 = 0$.

A. $\{-4, 3\}$

B. $\{3, 6\}$

C. $\{-2, 10\}$

D. $\{1, 8\}$

Handwritten solution for the quadratic equation $y^2 - 8y - 20 = 0$.

The equation is written as $y^2 - 8y - 20 = 0$. The terms $-8y$ and -20 are circled in orange.

A factoring table is shown with the following structure:

| | | |
|-------|--------|-------|
| | y | 2 |
| y | y^2 | $-8y$ |
| -10 | $-10y$ | -20 |


The terms $-8y$ and $-10y$ are circled in orange.

The factored form is written as $(y - 10)(y + 2) = 0$.

The solutions are derived from each factor:

- $y - 10 = 0 \implies y = 10$
- $y + 2 = 0 \implies y = -2$

The solutions are $y = 10$ and $y = -2$.


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LESSON 8-7 Solving $ax^2 + bx + c = 0$

Key Concept Factoring $ax^2 + bx + c$

Words To factor trinomials of the form $ax^2 + bx + c$, find two integers, m and p , with a sum of b and a product of ac . Then write $ax^2 + bx + c$ as $ax^2 + mx + px + c$, and factor by grouping.

Example

$$5x^2 - 13x + 6 = 5x^2 - 10x - 3x + 6$$

$$m = -10 \text{ and } p = -3$$

$$\begin{aligned} &= 5x(x - 2) + (-3)(x - 2) \\ &= (5x - 3)(x - 2) \end{aligned}$$

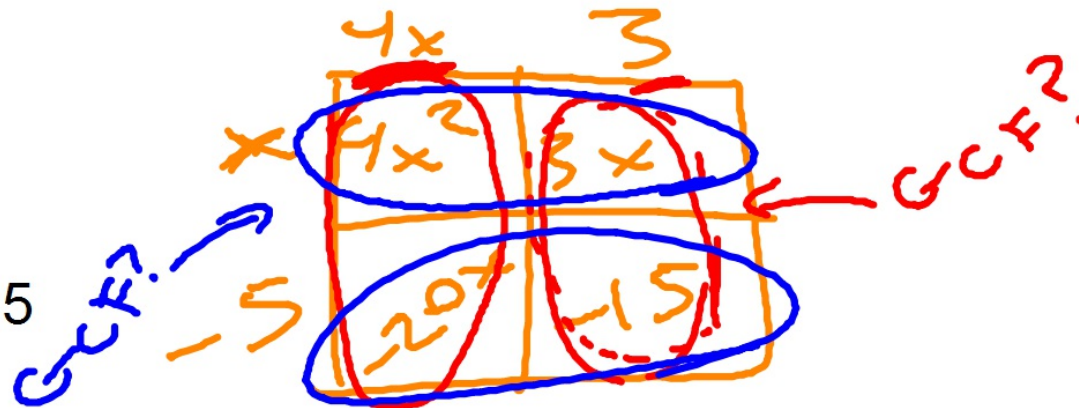
← This works, but let's try another way...

Challenge: how do we do this?

Let's try using the diamond (or x factor) method to approach this, BUT a little bit different....

$$(4x + 3)(x - 5)$$
$$4x^2 - 20x + 3x - 15$$
$$4x^2 - 17x - 15$$

first, let's multiply together....



...wouldn't it be nice to use the box method in reverse?

remember, to factor, we need to do all of this *in reverse*.

$$(4x + 3)(x - 5)$$

$$4x^2 - 20x + 3x - 15$$

$$4x^2 - 17x - 15$$

$$4x^2 - 17x - 15$$

NOW, let's try to factor...

1) Put the **middle number** on top.

2) Put the **PRODUCT** of the **first and last number** on the bottom.

$$\begin{array}{c} \text{ } \\ \diagdown \quad \text{-17} \quad \diagup \\ \text{-20} \quad \diagdown \quad \text{3} \\ \diagup \quad \text{-60} \quad \diagdown \\ \text{ } \end{array}$$

"what two numbers multiply to -60 and adds up to -17?"

$$(4x + 3)(x - 5)$$

$$4x^2 - 20x + 3x - 15$$

$$4x^2 - 17x - 15$$

| | | |
|----------|---------|--------|
| | $x - 5$ | |
| $4x + 3$ | $4x^2$ | $-20x$ |
| | $3x$ | -15 |

NOW, let's use the box method in reverse!

- 1) Put the **first and last terms** diagonal to each other.
- 2) put **the middle terms** in the other two boxes.
- 3) look across the rows and columns to factor appropriately!

Check Your Understanding



ns begin on page R13.



Examples 1–3 Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1. $3x^2 + 17x + 10$ $(3x + 2)(x + 5)$

2. $2x^2 + 22x + 56$ $2(x + 4)(x + 7)$

3. $5x^2 - 3x + 4$

4. $3x^2 - 11x - 20$ $(3x + 4)(x - 5)$

Check out the answer to number 2...they took out the GCF first!

①

$$3x^2 + 17x + 10$$

| | | |
|-----|--------|------|
| | $3x$ | 2 |
| x | $3x^2$ | $2x$ |
| 5 | $15x$ | 10 |

$$(3x^2 + 15x) + (2x + 10)$$
$$3x(x+5) + 2(x+5)$$

Check Your Understanding

 = Step-by-Step Solutions begin on page R13.



Examples 1–3 Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

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2. $2x^2 + 22x + 56$ $2(x + 4)(x + 7)$

3. $5x^2 - 3x + 4$ **prime**

4. $3x^2 - 11x - 20$ $(3x + 4)(x - 5)$

② $2(x^2 + 11x + 28)$
 $2(x + 4)(x + 7)$

| | | |
|-----|-------|------|
| | x | 4 |
| x | x^2 | $4x$ |
| 7 | $7x$ | 28 |

Check Your Understanding

 = Step-by-Step Solutions begin on page R13.

Examples 1–3 Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

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2. $2x^2 + 22x + 56$ $2(x + 4)(x + 7)$

3. $5x^2 - 3x + 4$ **prime**

4. $3x^2 - 11x - 20$ $(3x + 4)(x - 5)$

4

$\begin{array}{r} 4 \overline{) -11} \\ \underline{-60} \\ -20 \end{array}$
 $(3)(-20)$

$3x^2 + 4x - 15x - 20$

| | |
|--------|-------|
| $3x^2$ | $4x$ |
| $-15x$ | -20 |

x
 -5

Example 4

Solve each equation. Confirm your answers using a graphing calculator.

5. $2x^2 + 9x + 9 = 0$ $-\frac{3}{2}, -3$

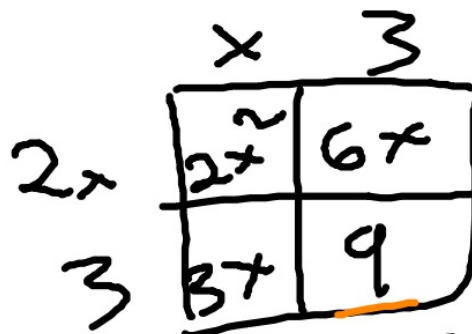
6. $3x^2 + 17x + 20 = 0$ $-\frac{5}{3}, -4$

7. $3x^2 - 10x + 8 = 0$ $\frac{4}{3}, 2$

8. $2x^2 - 17x + 30 = 0$ $\frac{5}{2}, 6$

1) factor

5



$(2x+3)(x+3) = 0$

2) zero product property!

$2x+3=0$
 $-3 \quad -3$

 $2x = -3$
 $\frac{2x}{2} = \frac{-3}{2}$
 $x = -\frac{3}{2}$

$x+3=0$
 $-3 \quad -3$

 $x = -3$

11. $(2x + 3)(x + 8)$ 13. $2(2x + 5)(x + 7)$

Practice and Problem Solving

Extra Practice is on page R8.

Examples 1–3 Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

10. $5x^2 + 34x + 24$

11. $2x^2 + 19x + 24$

12. $4x^2 + 22x + 10$ $2(2x + 1)(x + 5)$

13. $4x^2 + 38x + 70$

14. $2x^2 - 3x - 9$

15. $4x^2 - 13x + 10$ $(4x - 5)(x - 2)$

16. $2x^2 + 3x + 6$ **prime**

17. $5x^2 + 3x + 4$ **prime**

18. $12x^2 + 69x + 45$ $3(4x + 3)(x + 5)$

19. $4x^2 - 5x + 7$ **prime**

20. $5x^2 + 23x + 24$
 $(5x + 8)(x + 3)$

21. $3x^2 - 8x + 15$ **prime**

Handwritten work for problem 13:

⑬ ② $(2x^2 + 14x + 35)$

~~AC~~ $2x^2 + 5x + 14x + 70$

| | | |
|-----|--------|------|
| | $2x$ | 5 |
| x | $2x^2$ | $5x$ |
| 7 | $14x$ | 35 |

$2(2x + 5)(x + 7)$

15. $4x^2 - 13x + 10$ $(4x - 5)$ $(x - 2)$

~~$\begin{array}{r} -13 \\ -5 \quad -8 \\ \hline 40 \end{array}$~~

$4x^2 - 5x - 8x + 10$

x
 -2

| | | |
|------|--------|-------|
| | $4x^2$ | $-5x$ |
| x | | 10 |
| -2 | $-8x$ | |

Solve each equation. Confirm your answers using a graphing calculator.

23. $2x^2 + 9x - 18 = 0$ $\frac{3}{2}, -6$

24. $4x^2 + 17x + 15 = 0$ $-\frac{5}{4}, -3$

25. $-3x^2 + 26x = 16$ $\frac{2}{3}, 8$

26. $-2x^2 + 13x = 15$ $\frac{3}{2}, 5$

27. $-3x^2 + 5x = -2$ $-\frac{1}{3}, 2$

28. $-4x^2 + 19x = -30$ $-\frac{5}{4}, 6$

25 $-3x^2 + 26x = 16$
 $\quad\quad\quad -16 \quad -16$

$-(-3x^2 + 26x - 16) = 0$

$3x^2 - 26x + 16 = 0$

~~$\begin{array}{r} -26 \\ -24 \quad -2 \\ \hline 48 \end{array}$~~

$3x^2 - 24x - 2x + 48 = 0$
 $\quad\quad\quad x \quad -8$

| | |
|--------|--------|
| $3x^2$ | $-24x$ |
| $-2x$ | 48 |

$(x - 24)(3x - 2) = 0$

~~$x - 24 = 0$~~

~~$x = 24$~~

$3x - 2 = 0$
 $\quad\quad +2 \quad +2$

$\frac{3x}{3} = \frac{2}{3}$
 $x = \frac{2}{3}$

